To recognize the fundamental variability of the sea surface, it becomes necessary to treat the characteristics of the sea surface in statistical terms.

The term *irregular waves* will be used to denote natural sea states in which the wave characteristics are expected to have a statistical variability in contrast to *monochromatic waves*, where the properties may be assumed constant. **Monochromatic waves** may be generated in the laboratory but are rare in nature. “Swell” describes the natural waves that appear most like monochromatic waves in deep water, but swell, too, is fundamentally irregular in nature.

We note that the sea state in nature during a storm is always short-crested and irregular. Waves that have travelled far from the region of generation are called *swells*. These waves have a much more limited range of variability, sometimes appearing almost monochromatic and long-crested.

When the wind is blowing and the waves are growing in response, the sea surface tends to be confused: a wide range of heights and periods is observed and the length of individual wave crests may only be a wave length or two in extent (short-crested).

Such waves are called **wind seas**, or often, just sea. Long period waves that have travelled far from their region of origin tend to be more uniform in height, period, and direction and have long individual crests, often many wave lengths in extent (i.e., long-crested). These are termed **swell**. A sea state may consist of just sea or just swell or may be a <u>combination of both</u>.

The ocean surface is often a combination of many wave components. These individual components were generated by the wind in different regions of the ocean and have propagated to the point of observation. If a recorder were to measure waves at a fixed location on the ocean, a non-repeating wave profile would be seen and the wave surface record would be rather irregular and random.

Although individual waves can be identified, there is significant variability in height and period from wave to wave. Consequently, definitions of wave height, period, and duration must be statistical and simply indicate the severity of wave conditions.

The concept of ***significant wave height***, which has been found to be a very useful index to characterize the heights of the waves on the sea surface, will be introduced.

Peak period and ***mean wave direction*** which characterize the dominant periodicity and direction of the waves, will be defined.

However, these parameterizations of the sea surface in some sense only index how big some of the waves are. When using irregular wave heights in engineering, the engineer must always recognize that larger and smaller (also longer and shorter) waves

In the time-domain analysis of irregular or random seas, wave height and period, wavelength, wave crest, and trough have to be carefully defined for the analysis to be performed.

The definitions provided earlier in the regular wave is that the crest of a wave is any maximum in the wave record, while the trough can be any minimum.

However, these definitions may fail when two crests occur within an intervening trough lying below the mean water line.

The more common definitions of wave period are the time interval between successive crossings of the mean water level by the water surface in a downward direction called *zero down-crossing period* or *zero up-crossing period* for the period deduced from successive up-crossings.

First, it would be necessary to assume that the process described by the wave record (i.e., a sea state), say *η(t)*, is ***stationary***, which means that the statistical properties of *η(t)* are independent of the origin of time measurement. Since the statistics of stationary processes are time-invariant, there is no drift with time in the statistical behaviour of *η(t)*. The stationarity requirement is necessary as we shall see later for developing a *probability distribution* for waves, which is the fraction or percentage of time an *event* is not exceeded. The probability distribution may be obtained by taking *η1(t1), η2(t1), η3(t1),...,* as variables, independent of the instant *t1*. If in addition, *η(t)* can be measured at different locations and the properties of *η(t)* are invariant or do not depend on location of measurements, the process may then be assumed ***homogenous***. In reality, *η(t)* may be assumed stationary and homogenous only for a limited duration at the location data are gathered (usually wind waves 3 hr or less).

Second, the process *η(t)* is assumed to be ***ergodic***, which means that any measured record of the process say *η1(t)* is typical of all other possible realizations, and therefore, the average of a single record in an ensemble is the same as the average across the ensemble.

For an ergodic process, the sample mean from the ensemble approaches the real **mean *μ***, and the sample variance approaches the **variance *σ*** of the process (sea state). The ergodicity of *η(t)* implies that the measured realization of *η(t)*, say *η1(t1)* is typical of all other possible realizations *η2(t1), η3(t1),* ...., all measured at one instant *t1*.

The concept of ergodicity permits derivation of various useful statistical information from a single record, eliminating the need for multiple recordings at different sites.

The assumptions of stationarity and ergodicity are the backbones of developing wave statistics from wave measurements. It is implicitly assumed that such hypotheses exist in reality, and are valid, particularly for the sea state.

Two parameters are frequently used in the probability distribution for waves. These are the ***spectral width ν*** and the ***spectral bandwidth ε***, and are used to determine the narrowness of a wave spectra.

These parameters range from 0 to 1, and may be approximated in terms of spectral moments by

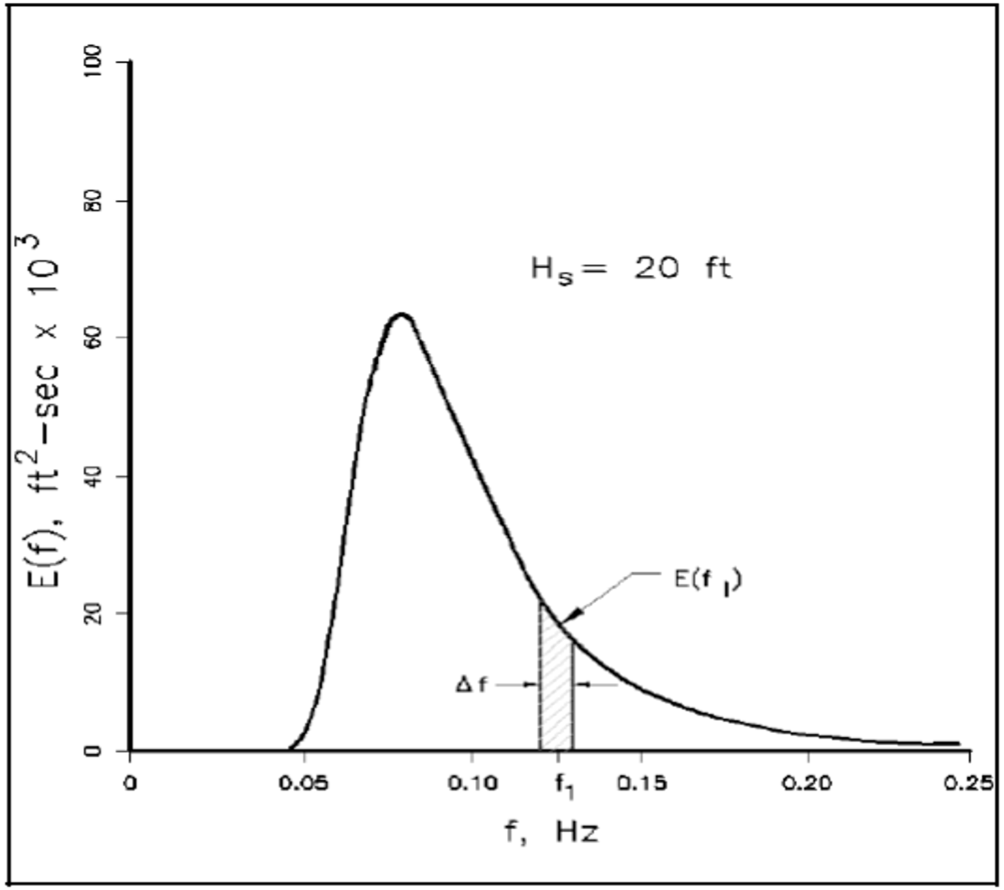

Figure II-1-37. Energy density and frequency relationship (Chakrabarti 1987)

$$\nu = \sqrt{\frac{m_0 m_2}{m_1^2} - 1}$$

$$\varepsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}} \qquad \text{(II-1-150)}$$

Spectrum on deep water

In a point P a sea state is registered obtaining $\eta = \eta(t)$, function that describes the sea surface

Assuming for $\eta(t)$ a Guassian distribution with mean value equal to μ_η it is possibile to describe the sea surface as a sum of elementary components, as

$$\eta(t) = \Sigma_n a_n \cos(\sigma_n t - \varepsilon_n)$$

The density of energy of the sea state is equal to

$$E = (\rho g / T) \int_{(o, T)} (\eta^2) \, dt$$

$$E = \rho g \int_{(0, \infty)} S(f) \, df$$

The momentum of order r of the spectrum S(f) is defined as

$m_r = \int_{(0, \infty)} f^r S(f) \, df$

and the momentum of order 0 m_o is given by

$m_o = \int_{(0, \infty)} S(f) \, df$

and consequently

$E = \rho g \, m_o$

An important parameter of the spectrum, introduced by Longuet-Higgins, is the width defined as

$\varepsilon_2 = \{(m_o m_2 / m_1^2) - 1\}^{0.5}$

The infinite narrow spectrum is characterized by a width parameter $\varepsilon_2 \rightarrow 0$.

JONSWAP (Jointh North Sea WAve Project) introduced by Hasselmann et al. (1973)

$$S(f) = \alpha\, g^2 (2\pi)^{-4} f^{-5} \exp\{-1.25 (f/f_p)^{-4}\} \exp\{\ln\gamma \exp[-0.5(f/f_p-1)^2/\omega^2]\}$$

where: α is the equilibrium parameter; γ is the amplification parameter and ω is the shape parameter.

For a stationary sea state, where the fetch gx/U^2 plays an important role, the above parameters have the following expressions

$\alpha = 0.076(gx/U^2)^{-0.22}$

$\gamma = 3.3$ (ranging from 1 to 7 usually)

$\omega = \omega_a = 0.07$ per $f \leq f_p$

$\omega = \omega_b = 0.09$ per $f > f_p$

$f_p = 3.5\, (gx/U^2)^{-0.33}\, g / U$

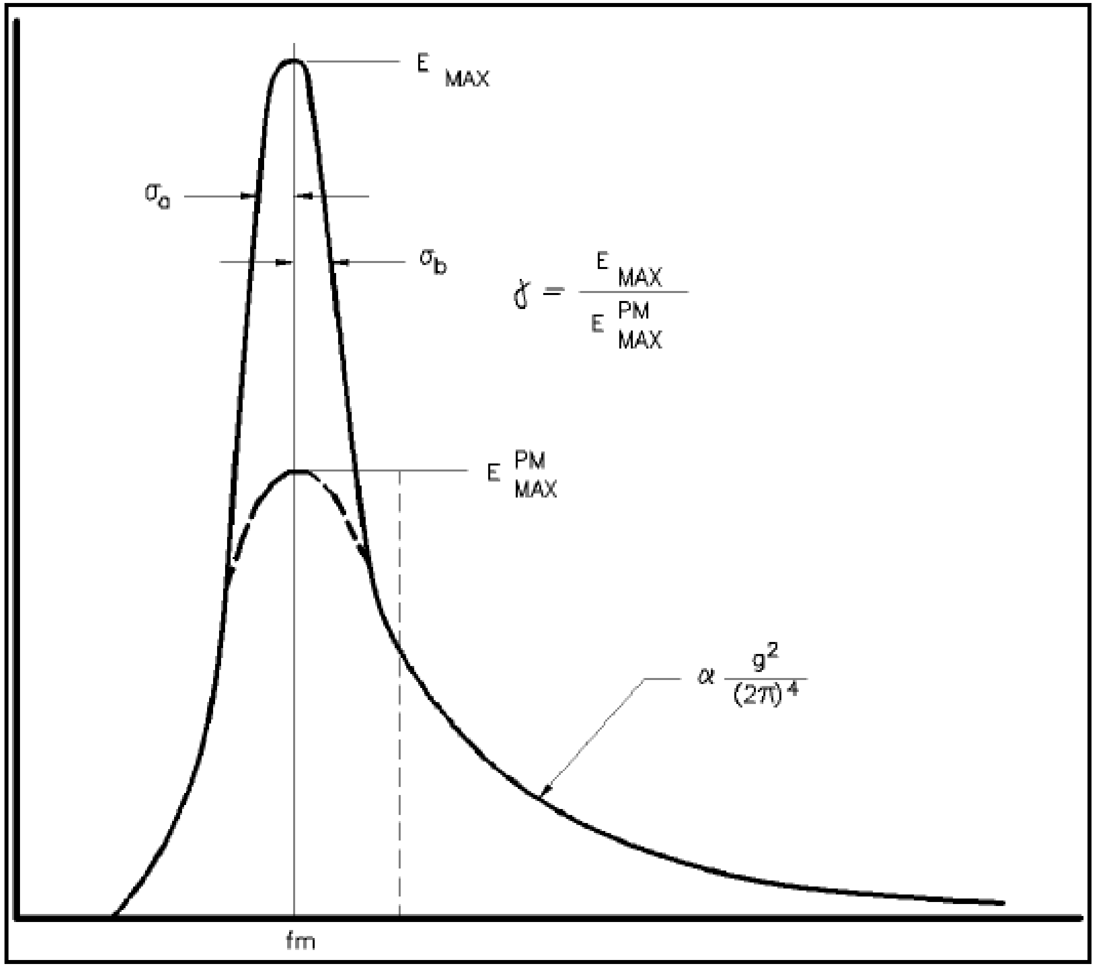

Figure II-2-22. Definition of JONSWAP parameters for spectral shape

$$H_S = 4.0\sqrt{m_0} \quad ; \quad H_{1/10} = 5.1\sqrt{m_0}$$

$$T_z = \sqrt{\frac{m_0}{m_2}} \quad ; \quad T_c = \sqrt{\frac{m_2}{m_4}}$$

$$\bar{\eta} = \sqrt{m_0} \quad ; \quad \varepsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}}$$

The equilibrium form of the *PM* spectrum for fully-developed seas may be expressed in terms of wave frequency f and wind speed U_w as

$$E(f) = \frac{0.0081g^2}{(2\pi)^4 f^5} \exp\left(-0.24 \left[\frac{2\pi U_w f}{g} \right]^{-4} \right)$$

In shallow water, the wave spectrum deviates from the standard spectra forms presented so far, and at frequencies above the peak, the spectrum no longer decays as *f-5*.

Kitaigorodoskii et al. (1975) showed that the equilibrium range is proportional to *-3* power of the wave number, and thus, the form of the spectrum is of *f-3* in the high-frequency range. This change is attributed to the effect of water depth on wave spectrum and to the interaction between spectral components.

The spectrum so obtained, the product of *JONSWAP* and the *Kitaigorodoskii depth function* accounting for the influence of the water depth, is called the *TMA spectrum* after the names of three sources of data used in its development (Texel, Marsen, and Arsloe).

The *TMA* spectrum was intended for wave hindcasting and forecasting in water of finite depth. This spectrum is a modification of the *JONSWAP* spectrum simply by substituting Kitaigorodoskii’s expression for effects of the finite depth equilibrium function. By using the linear wave theory, we find the following complete form of the *TMA* spectrum:

$$S_{TMA}(\omega,d) = S_{JONSWAP}(\omega)\ \Phi(\omega^*,d)$$

$$\Phi(\omega^*,d) = \frac{1}{f(\omega^*)}\left[1 + \frac{K}{\sinh K}\right] \quad ; \quad \omega^* = \omega\sqrt{\frac{d}{g}}$$

$$f(\omega^*) = \tanh^{-1}[k(\omega^*)d] \quad ; \quad K = 2\omega^{*2}\ f(\omega^*)$$

The wave spectra described so far have been one-dimensional frequency spectra. Wave direction does not appear in these representations, and thus variation of wave energy with wave direction was not considered. However, the sea surface is often composed of many waves coming from different directions.

In addition to wave frequency, the mathematical form of the sea state spectrum corresponding to this situation should therefore include the wave direction ϑ. Each wave frequency may then consist of waves from different directions ϑ. The wave spectra so obtained are *two-dimensional*, and are denoted by $E(f,\vartheta)$.

A mathematical description of the directional sea `state is feasible by assuming that the sea state can be considered as a **superposition of a large number** of regular sinusoidal wave components with different frequencies and directions.

With this assumption, the representation of a spectrum in frequency and direction becomes a direct extension of the frequency spectrum alone, allowing the use of *FFT* method.

It is often convenient to express the wave spectrum *E(F,ϑ)* describing the angular distribution of wave energy at respective frequencies by

$$E(f) = \int_{-\pi}^{\pi} E(f,\vartheta)\, d\theta$$

where the function *G(f,ϑ)* is a dimensionless quantity, and is known as the *directional spreading function*.

Other acronyms for *G(f,ϑ)* are the *spreading function*, *angular distribution function*, and the *directional distribution*.

From basic concepts of energy conservation and the fact that waves do attain limiting fully developed wave heights, it is obvious that wave generation physics cannot consist of **only wind source terms**.

There must be some physical mechanism or mechanisms that leads to a balance of wave growth and dissipation for the case of fully developed conditions.

Phillips (1958) postulated that one such mechanism in waves would be wave breaking. Based on dimensional considerations and the knowledge that **wave breaking has a very strong local effect on waves**,

Phillips argued that energy densities within a spectrum would always have a universal limiting value given by

$E(f) =(\alpha g^2 f^{-5})/(2 \pi)^4$

where E(f) is the spectral energy density in units of length squared per hertz and α was understood to be a universal (dimensionless) constant approximately equal to 0.0081.

It should be noted here that energy densities in this equation are proportional to f-5, and that they are independent of wind speed.

Phillips hypothesized that local wave breaking would be so strong that wind effects could not affect this universal level. In this context, a **saturated region of spectral energy** densities is assumed to exist in some region from near the spectral peak to frequencies sufficiently high that viscous effects would begin to be significant.

This region of saturated energy densities is termed the **equilibrium range** of the spectrum.

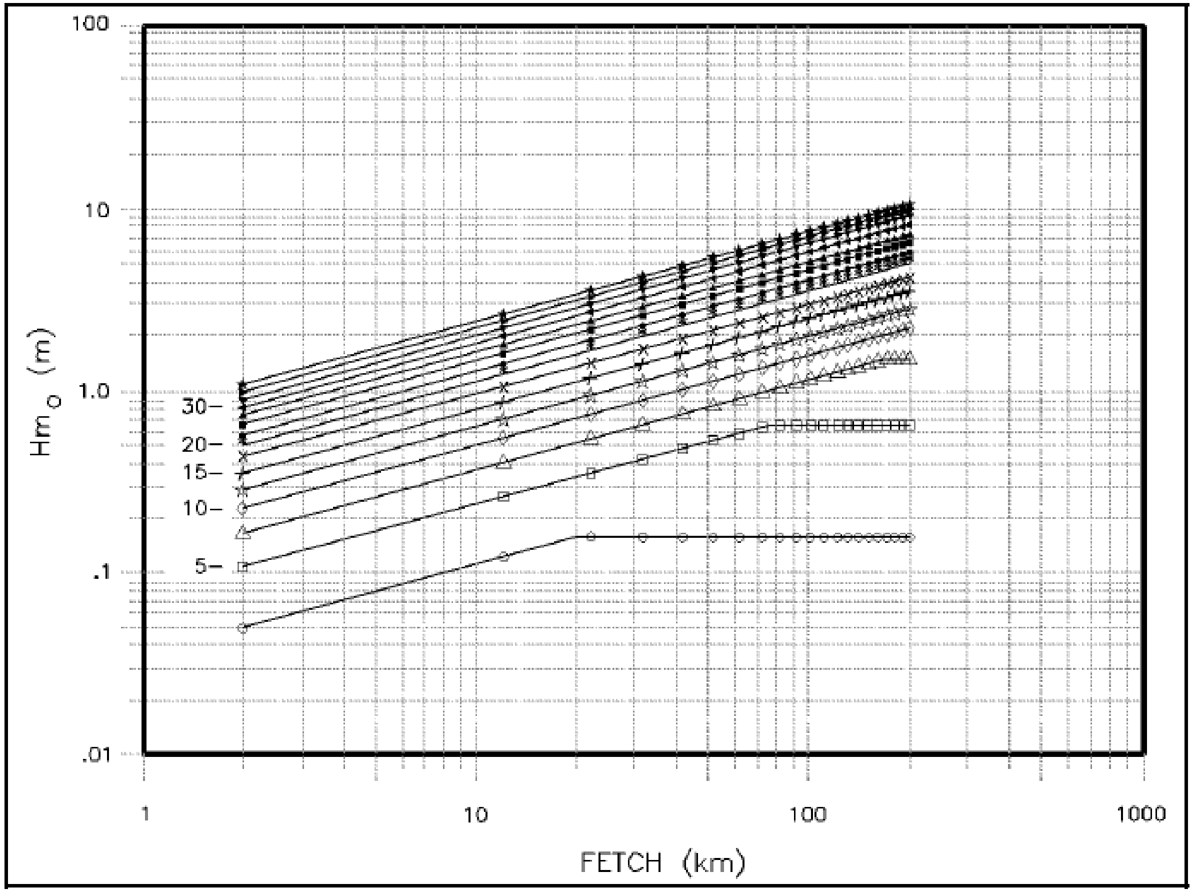

Figure II-2-23. Fetch-limited wave heights

A “fully developed” wave height would evolve under the action of the wind. From basic concepts of energy conservation and the fact that waves do attain limiting fully developed wave heights, it is obvious that wave generation physics cannot consist of only wind source terms.

There must be some physical mechanism or mechanisms that leads to a balance of wave growth and dissipation for the case of fully developed conditions. Phillips (1958) postulated that one such mechanism in waves would be wave breaking.

Based on dimensional considerations and the knowledge that wave breaking has a very strong local effect on waves

This region of saturated energy densities is termed the equilibrium range of the spectrum.

Kitaigorodskii (1962) extended the similarity arguments of Phillips to distinct regions throughout the entire spectrum where different mechanisms might be of dominant importance. Pierson and Moskowitz (1964) followed the dimensional arguments of Phillips and supplemented these arguments, with relationships derived from measurements at sea.

They extended the form of Phillips spectrum to the classical Pierson-Moskowitz spectrum

$$E(f) = (\alpha g^2 f^{-5} / (2\pi)^4) * \exp(0,74*(f/f_u)^{-4})$$

f_u = limiting frequency for a fully developed wave spectrum (assumed to be a function only of wind speed)

In the numerical models to generate sea state, it was recognized that waves in nature are not only made up of an infinite (continuous) sum of infinitesimal wave components at different frequencies but that each frequency component is made up of an infinite (continuous) sum of wave components travelling in different directions.

Thus, when waves travel outward from a storm, a single “wave train” moving in one direction does not emerge.

Instead, **directional wave spectra** spread out in different directions and disperse due to differing group velocities associated with different frequencies

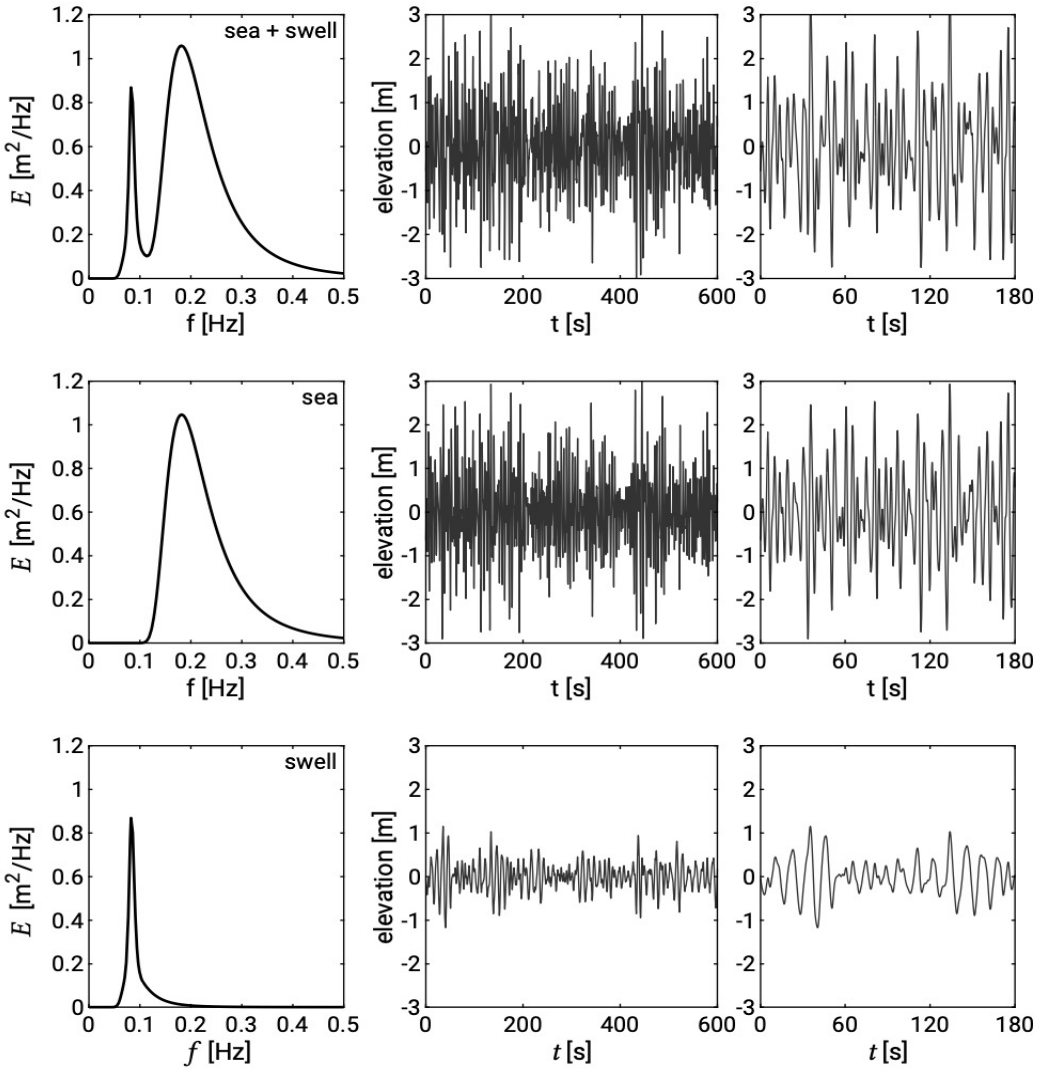

Figure 3.7: Surface elevation time series and corresponding spectra for sea and swell combined (upper panel), sea (middle panel) and swell (lower panel). To generate the sea component, a so-called JONSWAP spectrum (a typical spectrum for sea, see Sect. 3.5.1) is used with $H_S = 1.8$ m, $T_p = 5.5$ s and a peak enhancement factor $\gamma = 1$ (see eq. (6.3.15) in Holthuijsen, 2007). For the swell component, $H_S = 0.5$ m, $T_p = 12$ s and $\gamma = 5$ are used. For both sea and swell, the peak-width parameter is $\sigma = 0.07$ for $f \leq f_p$ and $\sigma = 0.09$ for $f > f_p$. The sea and swell spectra are combined in a bi-modal spectrum of swell and sea. The time series are generated from the spectra at a sampling frequency of 25 Hz assuming random phases. Note that the timeseries in the right panels correspond to Fig. 3.6.

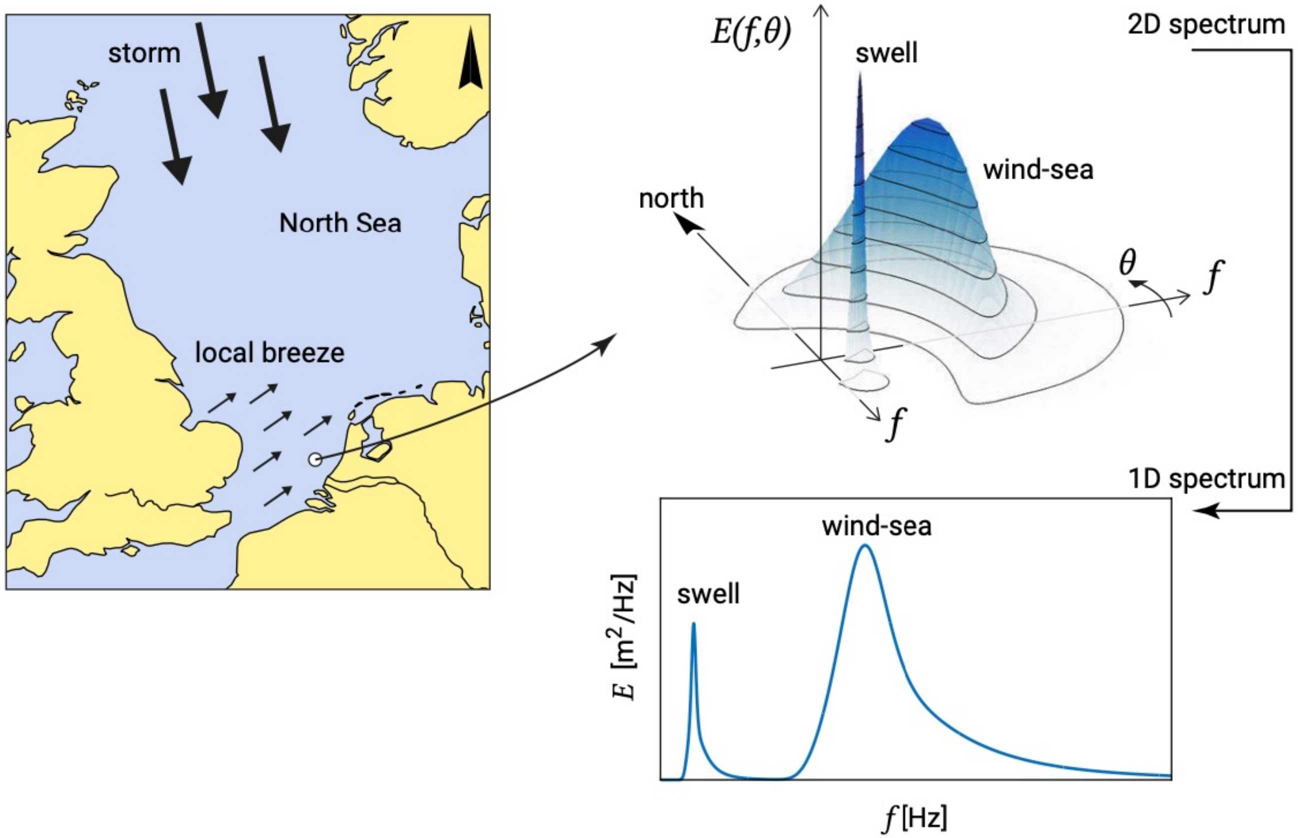

Figure 3.14: At a location off the Dutch coast, a northerly swell, generated by a storm off the Norwegian coast, meets a southwesterly sea generated by a local breeze (left). The 2D spectrum represents the spectral energy as a function of frequency and direction and is constructed by combining JONSWAP spectral shapes with Gaussian directional distributions. The 1D spectrum is obtained through integration over all directions. Even though in the 2D spectrum the swell peak is higher than the sea peak, the sea peak is the larger peak in the 1D spectrum because of the larger directional spreading for sea as compared to swell.